

HOMEWORK-8

Total Points: 65

1. [20 Points] Let's say node n in an AVL tree is imbalanced. The left and right subtrees of n are AVL trees. Node n is left heavy, and the left child of n is not heavy. Give an example for which if we perform an LR rotation on n and new_root is the parent of n after the rotation, then the subtree rooted at new_root is not an AVL tree. You can use a binary tree of integers in your example. The height of the tree in your example must be a constant, and all nodes must contain a value. Show how an LL rotation doesn't have this problem for the same example.
2. [15 Points] If every node in a binary tree has two or zero children and n is the total number of leaf nodes, then prove that the total number of internal nodes is $n-1$. You can use mathematical induction in your proof.
3. [10 Points] If there are n leaves in a nearly complete binary tree, what are all possible values for the total number of internal nodes in terms of n ? Justify your answer.
4. [5 Points] What are the minimum and maximum number of nodes in a max heap of height h ? Justify your answer.
5. [15 Points] Design an algorithm to identify the unique tree from a given preorder and inorder traversals. Instead of writing your algorithm, demonstrate how your algorithm works via the following example. Draw the unique tree from the following preorder and inorder traversals. Show all intermediate steps.

PREORDER: 1 7 9 8 10 12 13 14 18 19 20

INORDER: 7 8 9 10 1 14 19 18 20 13 12